Math 1314
Absolute Extrema

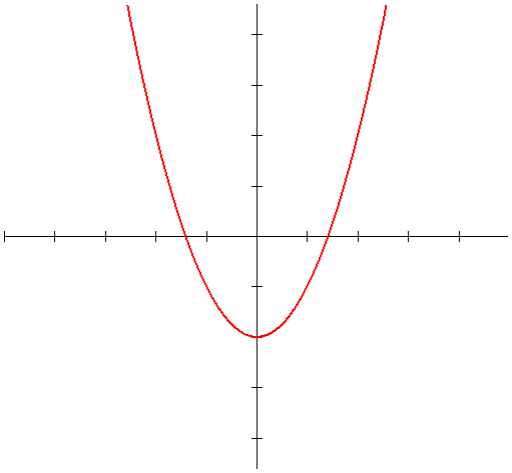
In earlier sections, you learned how to find relative (local) extrema. These points were the high points and low points relative to the other points around them. In this section, you will learn how to find **absolute extrema**, that is the highest high and/or the lowest low on the domain of the function, or on a specific closed interval.

Absolute Extrema on the Domain of *f*

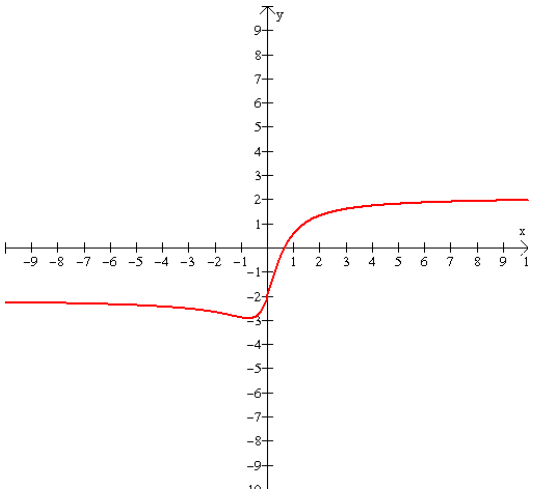
Definition: If $f(x) \le f(c)$ for all x in the domain of f, the $f(c)$ is called the **absolute maximum value** of f. If $f(x) \ge f(c)$ for all x in the domain of f, the $f(c)$ is called the **absolute minimum value** of f.

Sometimes you will be asked to find the absolute extrema over the interval $(-\infty, \infty)$.

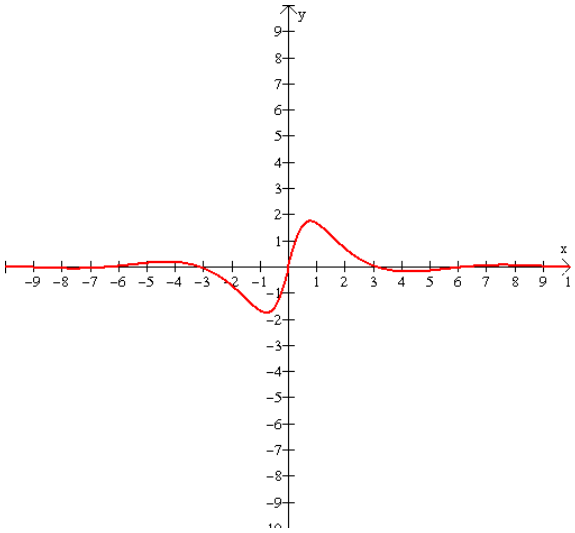
Example 1: State the absolute maximum and/or absolute minimum value(s).



Example 2: State the absolute maximum and/or absolute minimum value(s).



Example 3: State the absolute maximum and/or absolute minimum value(s).



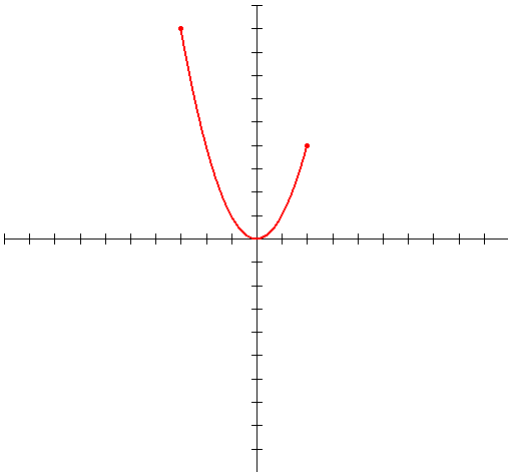
As you can see from these three examples, the absolute extrema may or may not exist. To find absolute extrema on $(-\infty,\infty)$ (or on the entire domain of the given function) algebraically, you must graph the function using the guide to curve sketching.

Absolute Extrema on a Closed Interval

More often you will be asked to find the absolute extrema over a closed interval [a, b], and, for a continuous function, those will always exist and they are much easier to find.

Theorem: If a function *f* is continuous on a closed interval [a, b], then *f* has both an absolute maximum value and an absolute minimum value on [a, b].

Example 4: Find the absolute extrema of $f(x) = x^2$ on the interval [-3, 5]



We can also find absolute extrema algebraically.

Finding the Absolute Extrema of *f* on a Closed Interval

1. Find the critical points of *f* that lie in (a, b).
2. Compute the value of the function at every critical point found in step 1 and also compute *f*(*a*) and *f*(*b*).
3. The absolute maximum value will be the largest value found in step 2, and the absolute minimum value will be the smallest value found in step 2.

Note: The absolute maximum value or the absolute minimum value refers to the *y* value of the point on the graph, or *f*(*x*). Read the question carefully!

Example 5: Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3 + 3x^2$ over the interval [-1, 1].

Example 6: Find the absolute maximum value and the absolute minimum value of the function $f(x) = 3x^4 + 4x^3$ on the interval [-1, 2].

Example 7: Find the *x* coordinate of the absolute extrema of the function $f(x) = 5x^{\frac{4}{5}} - 2x$ on the interval [-1, 1].

Example 8: A stone is thrown straight up from the roof of an 80-ft building. The height (in feet) of the stone at any time t (in seconds), measured from the ground is given by $h(t) = -16t^2 + 64t + 80$. What is the maximum height the stone reaches?

From this section, you should be able to

- Identify absolute extrema on $(-\infty, \infty)$ from a graph of f.
- Identify absolute extrema on $[a, b]$ from a graph of f.
- Find absolute extrema on $[a, b]$ algebraically
- Solve word problems having to do with absolute extrema