Math 1314
The Chain Rule

In this lesson, you will learn the last of the basic rules for finding derivatives, the chain rules.

Example 1: Decompose $h(x) = \left(4x^3 + 2x - 1\right)^5$ into functions $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$.

Rule 9: The Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Example 2: Find the derivative if $f(x) = \left(2x^4 + 7\right)^5$.

Example 3: Find the derivative if $f(x) = \sqrt{5x^2 + 1}$.

Example 4: Find the derivative if $f(x) = \dfrac{-9}{(3x^2 - 4)^5}$.

We can also apply the chain rule in problems involving the exponential function and the logarithmic function.

Rule 10: The Chain Rule (Exponential Function)

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

Example 5: Find the derivative: $f(x) = 5e^{2x}$.

Rule 11: The Chain Rule (Logarithmic Function)

Rule 10: $\frac{d}{dx}\left[\ln|f(x)|\right] = \frac{f'(x)}{f(x)}$, provided $f(x) > 0$

Example 6: Find the derivative: $f(x) = \ln(4x^2 + 1)$.

Sometimes it is helpful to use the properties of logarithms to simplify a problem before we find the derivative:

Example 7: Find the derivative: $f(x) = \ln\left(3x^2\right)$

Example 8: Find the derivative: $f(x) = \ln\left[\left(2x^2 + 1\right)^4\left(5x^2 - 1\right)^3\right]$

We can also use the chain rules together with either the product rule or the quotient rule.

Example 9: Find the derivative: $f(x) = x^3 e^{2x}$

Example 10: Find the derivative: $f(x) = x^5 \ln\left(x^2 - 7\right)$

Example 11: Find the derivative if $f(x) = x^2\left(3x^2 - 5\right)^3$.

Example 12: Find the derivative if $f(x) = \dfrac{\left(x^2 + 6\right)^5}{x^3}$

Logarithmic differentiation

For some functions, the rules we have learned aren't enough. One such case is the function $f(x) = 3^x$. We will need a technique called **logarithmic differentiation** to find the derivative of this function.

1. Take the natural log of both sides of the problem.
2. Use log properties to simplify.
3. Take the derivative of both sides of the resulting problem. You will have $\dfrac{y'}{y}$ on one side of the equation.
4. Multiply both sides of the problem by *y*. This will leave you with *y*' = RHS, which is the derivative.

Example 13: Find the derivative: $f(x) = 7^x$.

Example 14: Find the derivative: $f(x) = 2^{5x^2}$.

From this lesson, you should be able to

- Apply the chain rules to appropriate problems to find derivatives
- Use the chain rule, together with other rules, to find derivatives
- Use log properties to simplify log problems before finding derivatives
- Use logarithmic differentiation to find derivatives of problems such as Ex. 12 and Ex. 13