Are Your Students Understanding, or Simply Turning the Crank?

Jeff Morgan
Interim Associate Provost for Education and Technology Innovation
University of Houston
jmorgan@math.uh.edu

Shameless Advertising

- High School Math Contest – February 15, 2014 – http://mathcontest.uh.edu
- Cedric French and Christian Haich – "AP Readers Discuss the 2013 AB Free Response Questions" – November 16, 2013
- AP Calculus Help Material and Online Electronic Quizzes – http://online.math.uh.edu/apcalculus/
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Fundamental Questions

- Can your students describe and explain the fundamental concepts in calculus?
- Can your students use fundamental concepts from calculus to work problems, even if they have not seen them before?

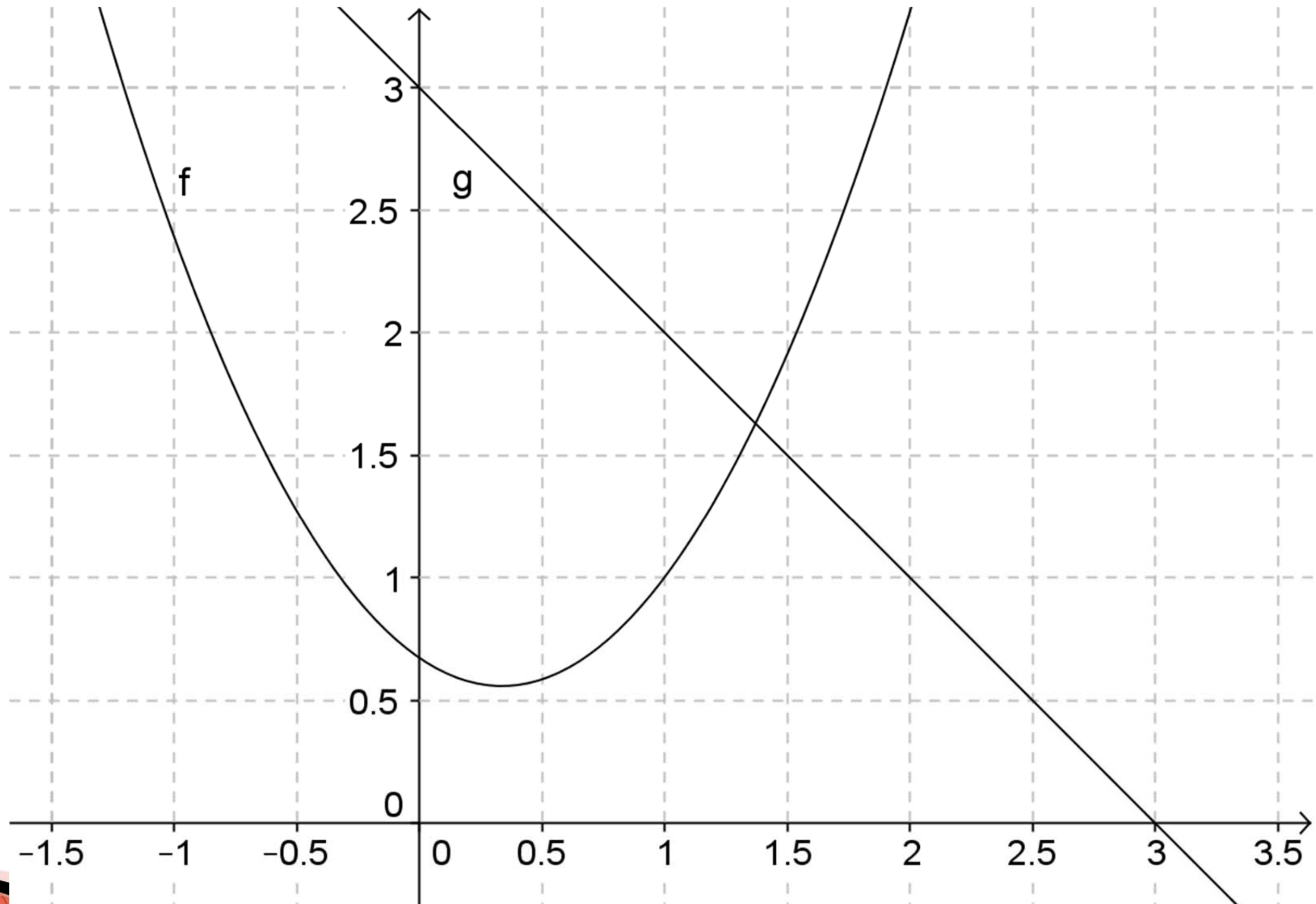
Instructions

Answer the following questions for each of the following problems.

1. What are the fundamental concepts associated with the problem?
2. What percentage of your students can successfully work the problem?
3. How will your "successful" students approach the problem?

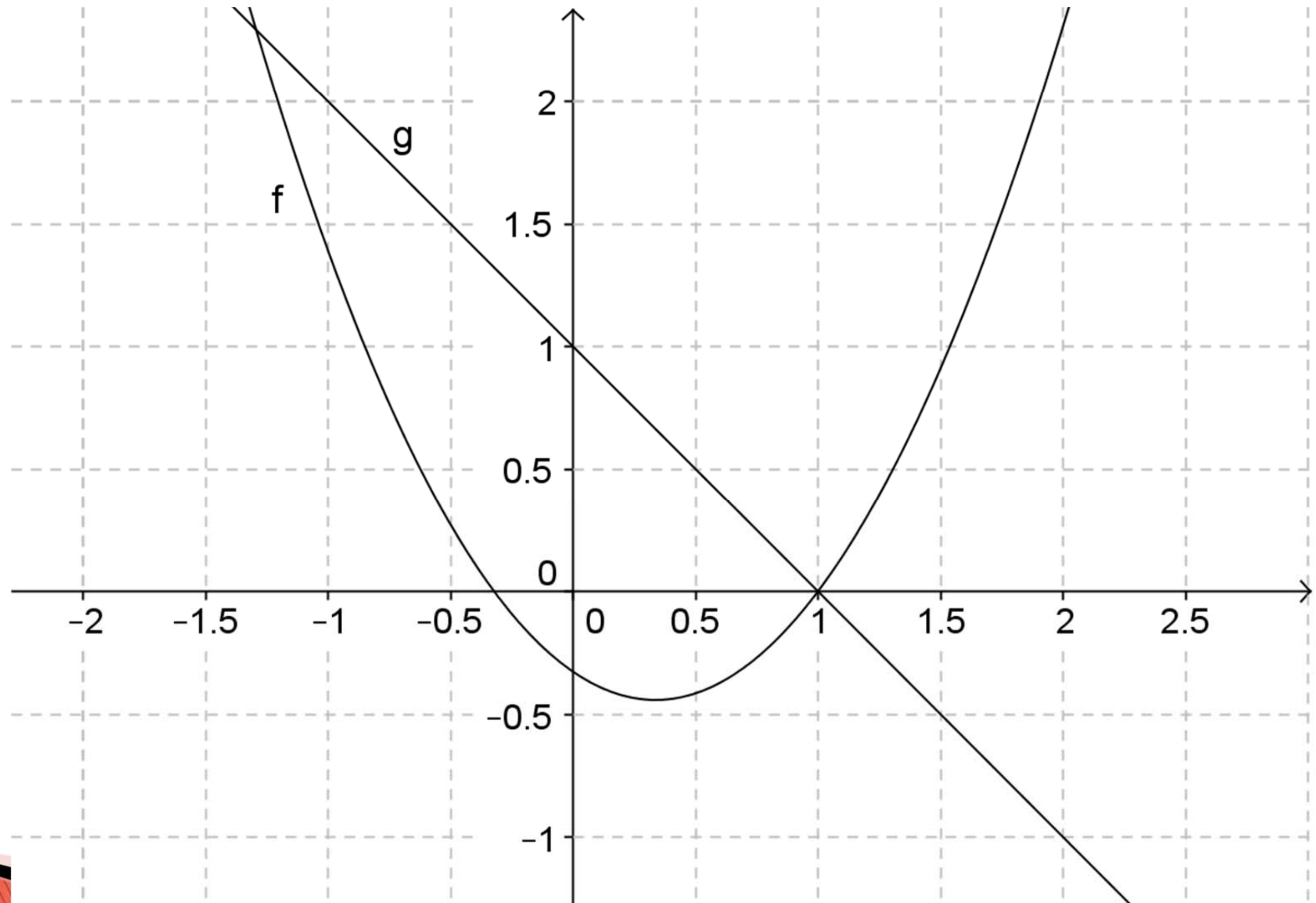
The functions f and g are graphed below.

$$\lim_{x \to 1}\left(2f(x) - 3g(x)\right) =$$



The functions f and g are graphed below.

$$\lim_{x\to 1} \frac{f(x)}{g(x)} =$$



The functions f and g are differentiable and have tangent lines at $x = 1$ given by $y = 2x - 1$ and $2x + 3y = 2$.

$$\lim_{x \to 1} \frac{f(x)}{g(x)} =$$

$$f(x) = \frac{2x^3 - 3x + 6}{x + 2}$$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} =$$

The functions f and g are differentiable. The tangent line to the graph of f at $x = 1$ is given by $y = -3x + 5$, and the tangent line to the graph of g at $x = 1$ is given by $2x + 3y = 4$. Let

$$H(x) = \frac{f(x)}{g(x)}$$

Give $H'(1)$.

True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

If f is an invertible function then

$$f^{-1}(x) = \frac{1}{f(x)}$$

f is an invertible differentiable function, and the tangent line to the graph of f at $x = 1$ is given by $y = -3x + 5$.

$$(f^{-1})'(2) =$$

True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

Suppose f is an invertible differentiable function. If $ax + by = c$ is a tangent line to the graph of f, then $ay + bx = c$ is a tangent line to the graph of f^{-1}.

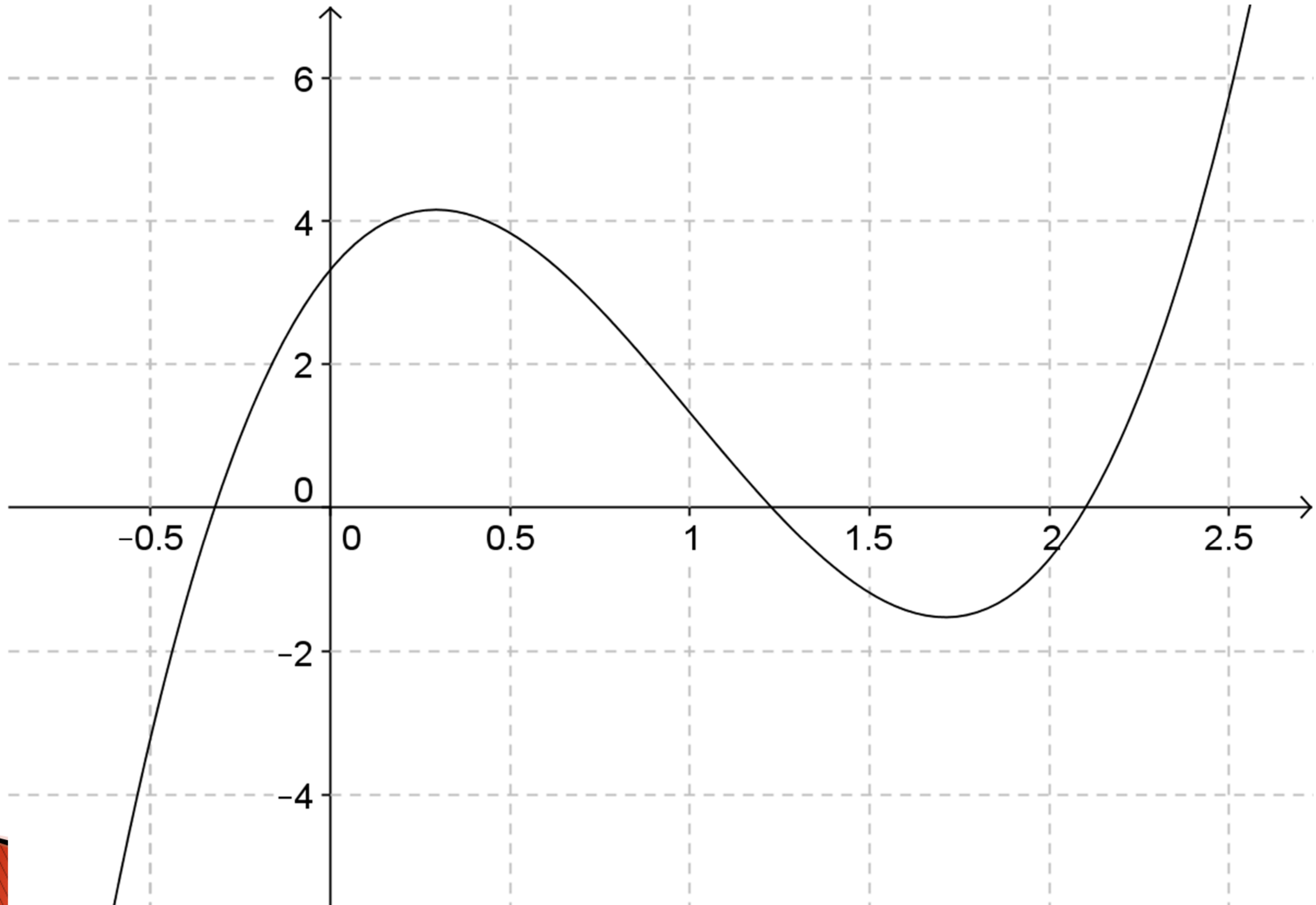
True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

If $f'(x) > 0$ when $x \neq 0$, then f is increasing.

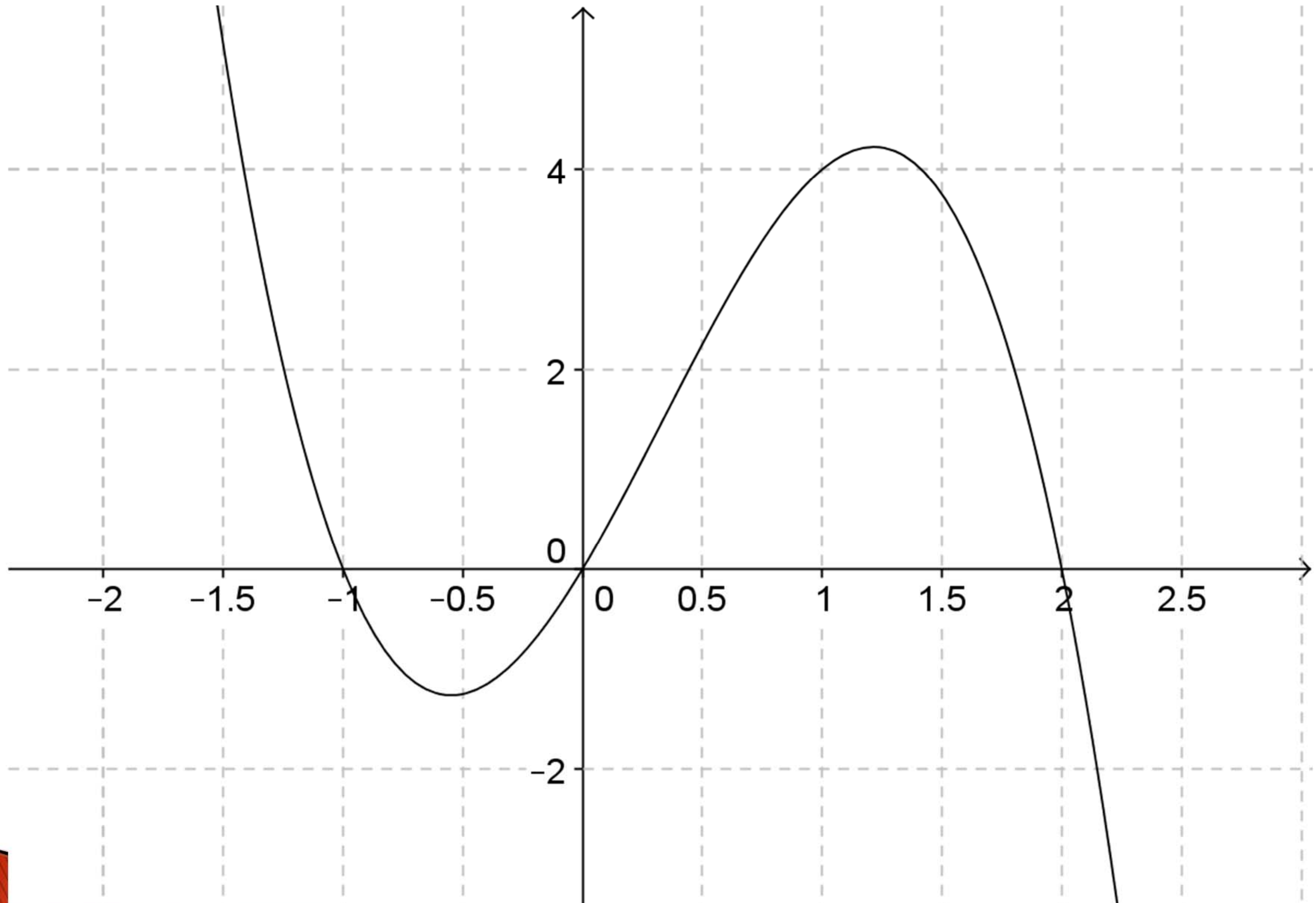
f is a differentiable function, $\lim_{x \to \infty} f(x) = \infty$, and $\lim\limits_{x \to -\infty} f(x) = \infty$. In addition, f has exactly one critical number. What can you conclude?

f is a differentiable function, $\lim_{x\to\infty} f(x) = \infty$, and $\lim\limits_{x\to-\infty} f(x) = \infty$. In addition, f has exactly 3 critical numbers a, b and c, with $a < b < c$. What are the possible classifications for a, b and c?

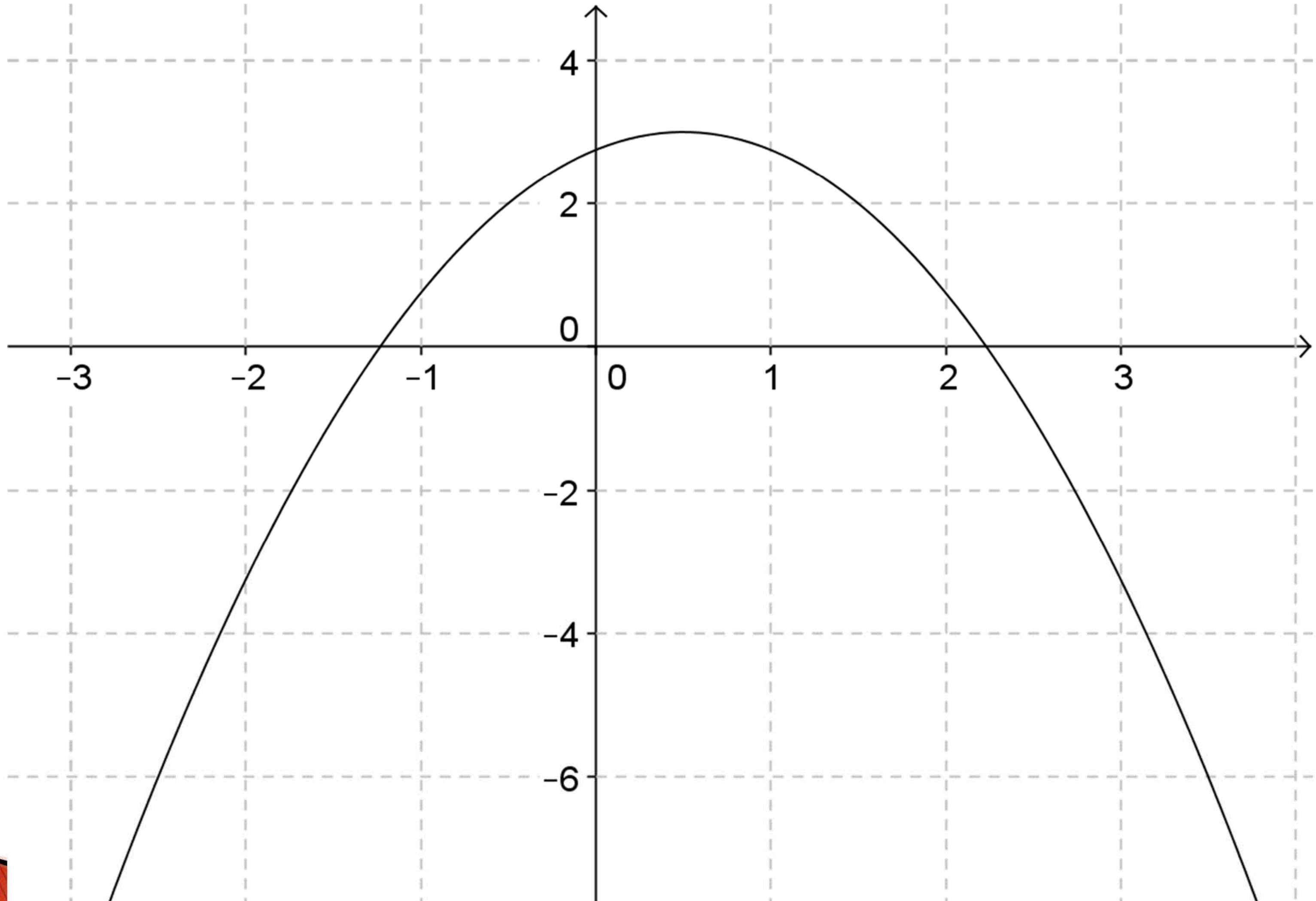
The graph of f' is shown below and $f(0) = 1$. Is f ever negative? Explain.



The graph of f' is given below. Where does the absolute maximum value of f occur?



The graph of f'' is shown below and $f'(0) = 1$. Give as much information as possible about the graph of f.



True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

If $f'(x) > 0$ for all $x > 2$, then

$$\lim_{x \to \infty} f(x) = \infty$$

True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

The graph of f' is shown. f is an increasing function.

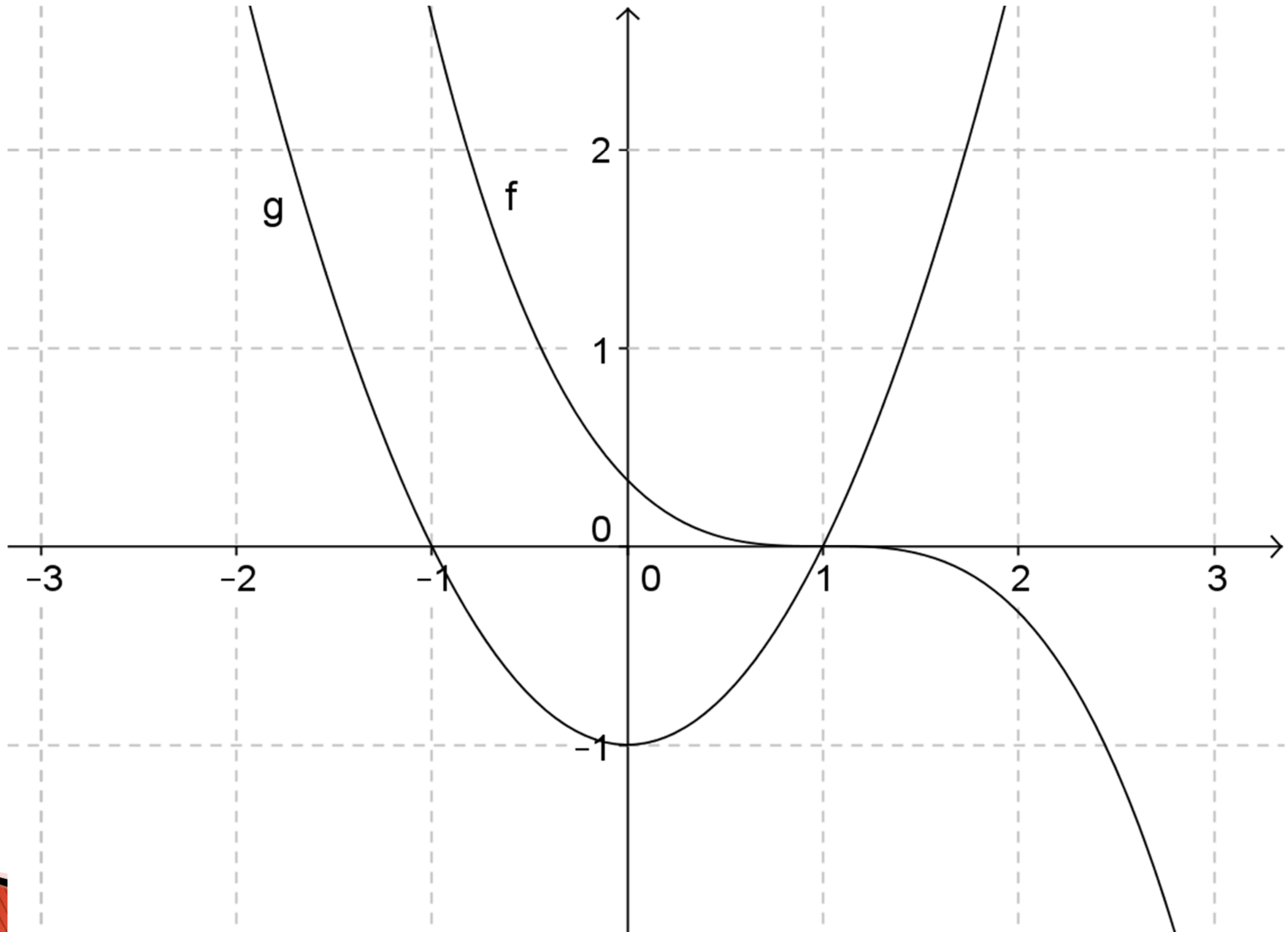
True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

If

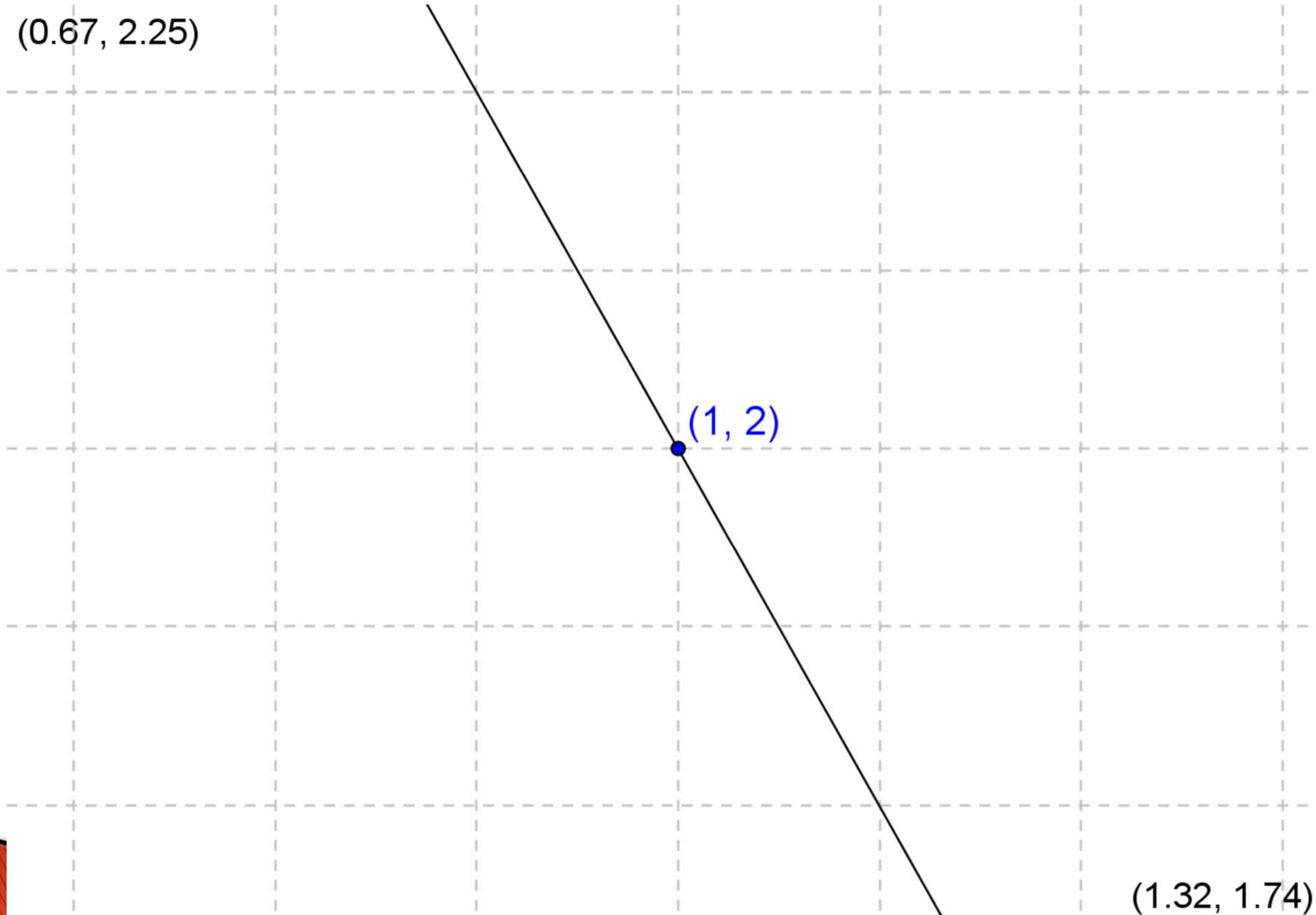
$$\lim_{h\to 0} \frac{f(1+h) - f(1-h)}{2h} = 0$$

then $f'(1) = 0$.

The graphs of f and g are given. Give the vertical asymptotes of $H(x) = \dfrac{f(x)}{g(x)}$.

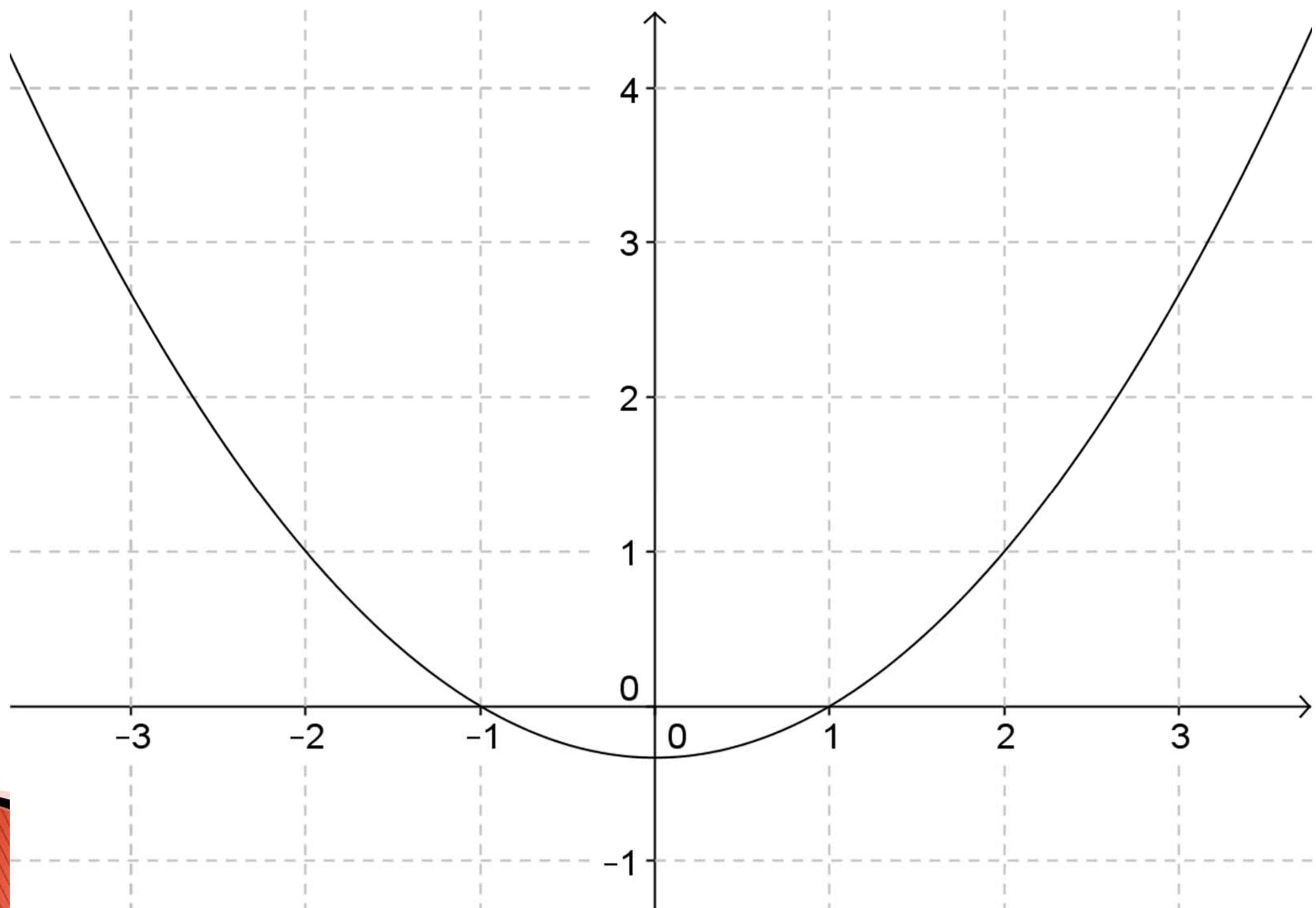


When we zoom in on the graph of f near $x = 1$ we see the picture below. What does this information tell us?



The tangent line to the graph of f at $x = 1$ is $y = 2x - 1$. Use Newton's method with a guess of $x = 1$ to approximate a root of f.

The graph of f is shown below. We can see there is a root of f at 1. Suppose we don't notice this. Show graphically how Newton's method with a guess of $x = 2$ will approximate this root.



Suppose f is an invertible differentiable function, and the tangent line to the graph of f^{-1} at $x = 1$ is given by $y = 3x - 2$. Give the tangent line to the graph of f at $x = 1$.

Suppose f is a differentiable function, and the tangent line to the graph of f at $x = 1$ is given by $y = -2x + 3$. Give the tangent line to the graph of $g(x) = f(2x^2 - 1)$ at $x = 1$.

Give a continuous function H so that

$$\lim_{x\to\infty} H(x) = 0$$

and

$$\sum_{n=1}^{\infty} H(n)(-1)^n$$

diverges.

Suppose

$$S(N) = \sum_{n=1}^{N} \frac{1}{n}$$

for each positive integer N. Describe the graph of $(N, S(N))$.

What does it mean to say that the series

$$\sum_{n=1}^{\infty} a_n$$

is conditionally convergent?

True or False. If the answer is True, then give an explanation. If the answer is False, then give a counter example.

Suppose f and g are nontrivial polynomials,

$$h(x) = \frac{f(x)}{g(x)}$$

and

$$\lim_{x \to \infty} h(x) = 0$$

Then h is either eventually decreasing or eventually increasing.

Part 1: State the alternating series test.

Part 2: Suppose f and g are polynomials,

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$$

and $g(x)$ is not zero for any positive integer values of x. Determine whether the alternating series test can be applied to the series

$$\sum_{n=1}^{\infty} \frac{f(n)}{g(n)} (-1)^n$$

Does the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{n^3 - 3n^2 + 4n - 7}{2n^4 + 3n^2 - n + 2}(-1)^n$$